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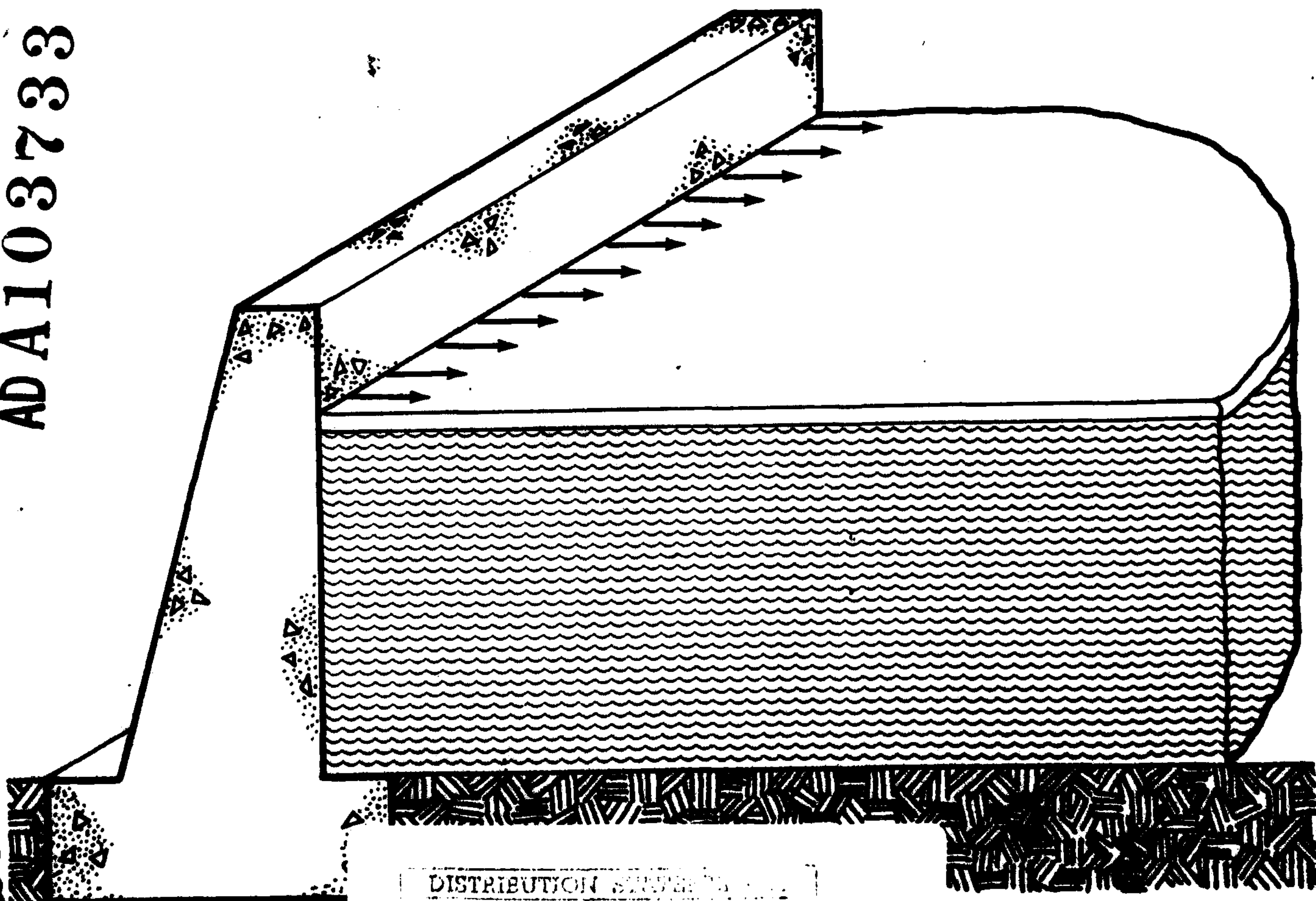

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On the buckling force of floating ice plates

Arnold D. Kerr

June 1981

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The calculation of the largest horizontal force a relatively thin floating ice plate may exert on a structure requires the knowledge of the buckling load for this floating plate. In the published literature on the stability of continuously supported beams and plates, it is usually assumed that this buckling force corresponds to the lowest bifurcation force p_{cr}. However, recent studies indicate that, generally, this is not the case, and this report clarifies the situation for floating ice plates. This problem is first studied on a simple model that exhibits the buckling mechanism of a floating ice plate but is amenable to an exact nonlinear analysis. This study shows that, depending on the ratio of the rigidities of the "liquid" and "plate," the post-buckling branch may rise or drop away from the bifurcation point. Thus, p_{cr} may or may not be the actual buckling load. It is also shown that when lift-off of "plate" from the "liquid" takes place, the

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20. Abstract (cont'd).

actual buckling load may drop substantially. This study is followed by an analysis of a floating compressed semi-infinite plate with a straight free edge, assuming that there is no lift-off. It is found that for this case there always exists a buckling load that is lower than p_{cr}. According to the obtained results, the value p_{cr} should be used with caution as a buckling load for floating ice plates. It is suggested that the buckling load be determined using the post-buckling equilibrium branch of the plate, taking into consideration the possibility of lift-off of the ice cover from the liquid base.

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PREFACE

This report was prepared by Dr. Arnold D. Kerr, of the Department of Civil Engineering, University of Delaware. Funding for this research was provided by U.S. Army Corps of Engineers Civil Works Project CWIS 31359, *Determination of the Uplifting Forces on Shore Structures.*

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ON THE BUCKLING FORCE OF FLOATING ICE PLATES

Arnold D. Kerr

INTRODUCTION

The analytical determination of the largest horizontal force a floating ice cover may exert on a structure is based on the criterion that an *ice force cannot be larger than the force capable of breaking up the ice cover.*

In many publications, as discussed by Korzhavin (1971) and Michel (1970), this failure criterion is related to a "crushing strength" of the ice cover. There it is assumed that the failure mechanism consists of the crushing and/or splitting of the floating ice plate in the immediate vicinity of the structure.

However, in a number of laboratory and field tests it was observed that for ice covers that are relatively thin (compared to the width of contact of plate and structure) the ice cover failed by buckling in the vicinity of the contact area. Thus, for these cases, the buckling force is smaller than the crushing force, and the determination of the largest force should be based on the force at which the floating ice cover is expected to buckle.

The related buckling analyses and tests were recently reviewed and discussed by Kerr (1978). In all these analyses it was assumed that the buckling load corresponds to the first bifurcation load p_{cr}, an assumption justified when the post-buckling equilibrium branch rises monotonically with increasing displacements. However, according to recent studies by Lekkerkerker (1962), Kerr (1972), and Plaut (1978), this is not the case in general. The purpose of the present report is to clarify this situation for the floating ice plate problem.

SIMPLE MODEL FOR ICE COVER BUCKLING

To study the basic features of the title problem we consider first the simple model shown in Figure 1, which consists of "rigid bars" interconnected by elastic spiral springs and resting on straight vertical springs. This model contains the observed buckling mechanism of a compressed floating ice cover and is amenable to a simple nonlinear exact analysis. It is assumed that the rigid bars are horizontal when subjected to the uniform load distribution q_0 (their own weight). In this position of equilibrium the spiral springs are stress-free and each of the straight vertical springs is compressed by a force kw_0. Thus, the vertical displacement is

$$w_0 = q_0 L/k \tag{1}$$

where k is the stiffness of the vertical springs. Next, the bars are subjected to an axial force P. For increasing P, deformed states of equilibrium may exist, as shown in Figure 2a. Assuming that the bars at joint 2 will not separate from the base spring and thus that

$$w_0 \gtrless L \sin\theta \quad \text{or} \quad q_0/k \gtreqless \sin\theta \tag{2}$$

the equilibrium equation for the deformed state is

$$P^* \sin\theta = \theta + k^* \sin\theta \cos\theta \tag{3}$$

where, denoting the parameter of the spiral springs by s,

$$P^* = PL/3s; \qquad k^* = kL^2/6s. \tag{4}$$

Note that q_0 does not appear in equilibrium equation 3 because of the linearity of the base springs. Also note that eq 3 is satisfied for $\theta = 0$. Thus, the straight state is in equilibrium (but not necessarily stable) for any load P. For very small values of θ, eq 3 reduces to

$$(P^* - 1 - k^*)\theta = 0. \tag{3'}$$

Thus, at the bifurcation point

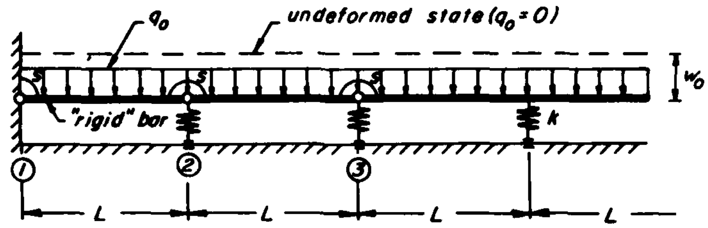

Figure 1. Simple buckling model (q_0 is the uniform load distribution, s the spiral spring parameter, and L the distance between the vertical springs).

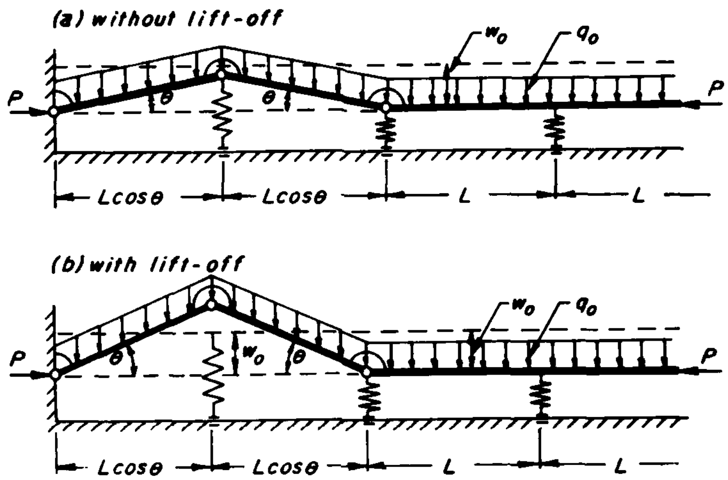

Figure 2. Deformed model.

$$P^*_{cr} = 1 + k^*. \tag{5}$$

Equation 3 was evaluated numerically for various values of k^*. The results are shown in Figure 3.

For a proper interpretation of the results shown in Figure 3a, note that, as shown in Figure 4 (Kerr 1970), the equilibrium branch rises monotonically when the model bars are constrained only by a spiral spring (bending effect), whereas the equilibrium branch drops when the model bars are constrained by a straight spring (effect of base). This is the reason why in Figure 3a, for large values of k^* (i.e. when the base stiffness predominates), the equilibrium branch drops at first and then rises, exhibiting a lower buckling load $P_L < P_{cr}$.

From the graphs in Figure 3a, it follows that the value of k^* which separates the different responses is located in the interval $0 < k^*_s < 0.5$. Using the perturbation method, Kerr (1972) showed that for the model under consideration

$$k^*_s = {}^1/_3. \tag{6}$$

Consider the model with $k^* = 1.0$. According to Figure 3, for each $P < P_L$ there exists only one straight equilibrium state, for $P_L < P < P_{cr}$ there exist five equilibrium states, and for each $P > P_{cr}$ three states of equilibrium are possible. It may be shown (Kerr 1974) that a straight equilibrium state is stable below P_{cr} and unstable above it, that the equilibrium states on branch $P_{cr}L$ are unstable, and that the equilibrium states on branch LB are stable.

When the axial load P has a small vertical eccentricity, the equilibrium branches are as shown by the dashed lines in Figure 3b. The effect of vertical shape imperfections is similar. Note that with increasing small eccentricities, or shape imperfections, the upper buckling load P_u drops rapidly. Structures of this type are referred to in the literature as "imperfection sensitive."

Because of the existence of load and geometrical imperfections (and dynamic inputs) in an actual situation, buckling will take place for $P_L < P < P_{cr}$, where P_L is the value for the structure with imperfections. Thus, for the cases that exhibit a P_L value, P_{cr} is not the true buckling load.

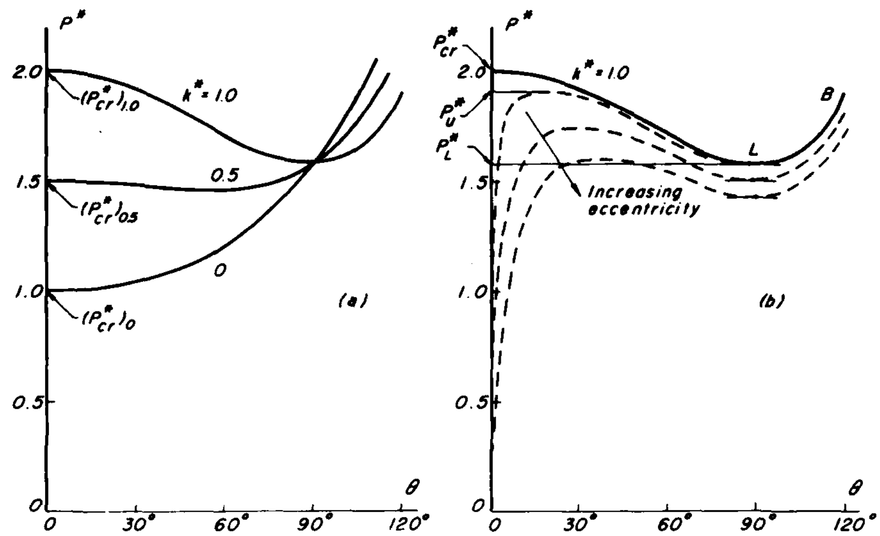

Figure 3. Equilibrium branches for model without lift-off. For θ < 0 the branches are symmetrical to the P axis.*

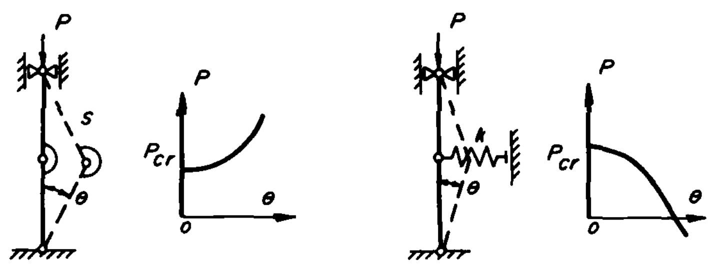

Figure 4. Simple models and their equilibrium branches.

Next consider the case when the bars are resting on, but are not attached to, the base spring at joint 2; i.e. they may lift off the spring base, as shown in Figure 2b. This may occur in an actual situation with floating ice covers. For this case, until lift-off takes place, the governing equilibrium equation is eq 3. After lift-off, namely when

$$w_0 < L \sin\theta \quad \text{or} \quad q_0/k' < \sin\theta \tag{7}$$

the equilibrium equation is

$$P^* \sin\theta = \theta + q^* \cos\theta \tag{8}$$

where

$$P^* = PL/3s; \quad q^* = q_0 L^2/6s. \tag{9}$$

The corresponding equilibrium branches for $k^* = 0.2$, 1.0 and a range of q^* values are shown in Figure 5.

The reason for the large post-buckling deformations in Figures 3 and 5 is the assumption that an axial force P remains the same also at the post-buckling equilibrium points. However, it was shown by Kerr (1973) that, when the axial compression force is induced by a temperature raise, the axial force drops and the post-buckling deformations are much smaller.

According to Figure 5a the admissibility of lift-off substantially reduces the P_L value, especially for small values of q^*. The resulting equilibrium branch for $q^* = 0.1$ is shown as a solid line. Note that the corresponding P_L^* equals 1.2; thus this value is about 25% lower than the P_L^* for the case when lift-off is not allowed.

Figure 5b shows that the admissibility of lift-off creates a P_L smaller than P_{cr}, even for an equilibrium branch that is monotonically increasing with no lift-off. This is an example where a stability analysis of the bifurcation point on the undeformed branch (for example, the Koiter method) is not suitable for predicting whether P_L exists.

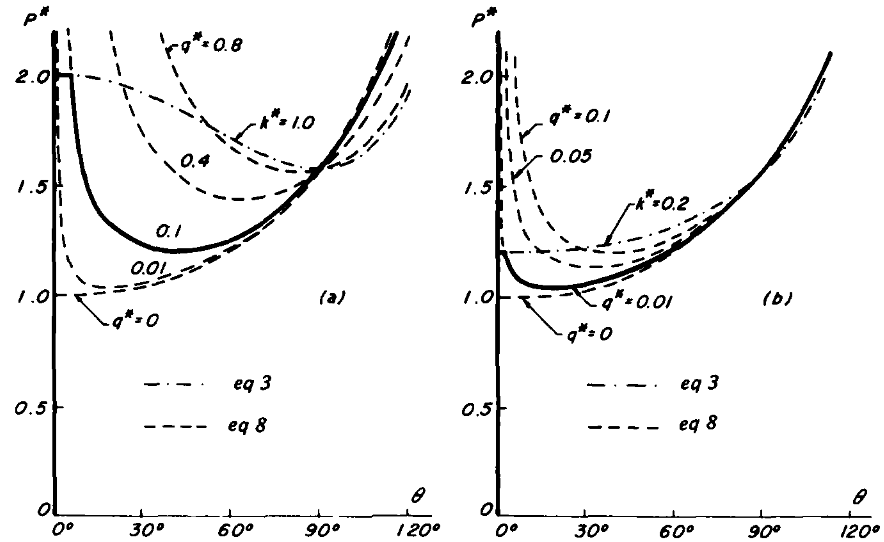

Figure 5. Equilibrium branches for model with lift-off.

The findings of this section suggest that for compressed floating ice plates, even in the absence of lift-off between plate and liquid base, a p_L smaller than p_{cr} may exist, as shown in Figure 3a. The results in Figure 5 suggest that the possibility of lift-off may greatly affect the post-buckling equilibrium branches and thus also the lower buckling loads p_L. Occurrence of lift-off may even create a p_L value where one does not exist when lift-off is not allowed.

To establish whether floating ice plates do respond in this way requires the solution of the nonlinear equations for plates on a liquid base. The study of all these phenomena is beyond the scope of the present report. Therefore, in the remainder of this report only the behavior of the equilibrium branches of compressed floating ice plates in the vicinity of p_{cr} (as shown in Figure 3a) is investigated in order to establish if, and under what conditions, a p_L smaller than p_{cr} will exist in the absence of lift-off.

SEMI-INFINITE FLOATING PLATE WITH FREE EDGE

The problem under consideration is shown in Figure 6. Because the load p (per unit length of plate boundary) is assumed to be constant, the plate displacements will be functions of x only. The vertical displacement component of a point (x, y) on the middle plane is assumed in the form $w_0 + w(x)$; w_0 = constant is the rigid body displacement due to the weight of plate q_0 (per unit area), and $w(x)$ is caused by bending deformations. Thus, $q_0 = \gamma w_0$. Following the derivations in Kappus (1939) or Kerr (1972), using Lagrange coordinates and neglecting the extensibility of the middle plane, the differential equation for $w(x)$ is

$$\frac{w''''}{1-w'^2} + \frac{4w'w''w''' + w''^3}{(1-w'^2)^2} + \frac{4w'^2 w''^3}{(1-w'^2)^3} + \frac{p}{D}\frac{w''}{(1-w'^2)^{3/2}} + \frac{\gamma}{D} w = 0 \qquad (10)$$

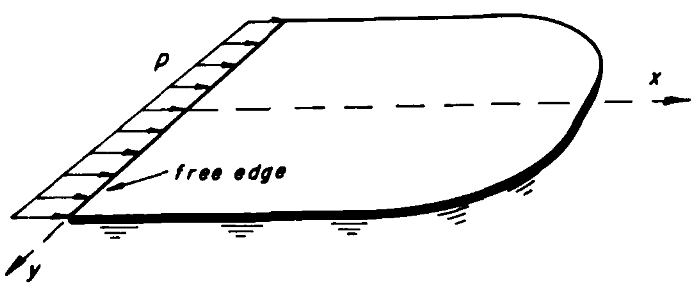

Figure 6. Semi-infinite plate with free edge.

where $D = Eh^3/12(1-\nu^2)$ is the flexural rigidity of the plate, γ is the base modulus, and $(\ )' = d(\ )/dx$. The boundary conditions are

$$w''(0) = 0 \tag{11}$$

$$\left|D\left(\frac{w''}{\sqrt{1-w'^2}}\right)' + pw'\right]_{x=0} = 0. \tag{12}$$

The formulation is highly nonlinear. In the mathematical literature there are no methods, as yet, for obtaining an exact closed form solution for this boundary value problem. Therefore, in the following, we analyze the post-buckling equilibrium branch in the vicinity of p_{cr} in order to establish if p_L exists. This is done using the perturbation method (Kerr 1972).

Following Keller (1968) the variables p and $w(x)$ are expanded in a Taylor series:

$$p(\epsilon) = p(0) + p_\epsilon(0)\epsilon + p_{\epsilon\epsilon}(0)\ \epsilon^2/2!\ + p_{\epsilon\epsilon\epsilon}(0)\ \epsilon^3/3!\ + \ldots$$

$$w(x,\epsilon) = w(x,0) + w_\epsilon(x,0)\epsilon + w_{\epsilon\epsilon}(x,0)\ \epsilon^2/2! + w_{\epsilon\epsilon\epsilon}(x,0)\ \epsilon^3/3!\ + \ldots \tag{13}$$

where ϵ is a small parameter. Denoting symbolically the nonlinear formulation consisting of eq 10-12 by

$$F[w(x), p] = 0, \tag{14}$$

it then follows, because of eq 13, that $F = F(\epsilon)$ only. Assuming that F is differentiable, we write

$$F(\epsilon) = F(0) + F_\epsilon(0)\epsilon + F_{\epsilon\epsilon}(0)\ \epsilon^2/2!\ + \ldots = 0. \tag{15}$$

Since ϵ is an arbitrary parameter, the above equation is satisfied when

$$F(0) = 0;\quad F_\epsilon(0) = 0;\quad F_{\epsilon\epsilon}(0) = 0; \ldots \tag{16}$$

These are the equations for the determination of the coefficients in eq 13.*

It should be noted that in eq 13 the term $w(x,0)$ represents the pre-buckling state. Hence

$$w(x,0) \equiv 0. \tag{17}$$

The formulation which corresponds to the first condition in eq 16, $F(0) = 0$, is satisfied for any point on the equilibrium branch with $w(x,0) \equiv 0$.

*Equation 15 may also be obtained by substituting eq 13 into 14 and then by grouping terms of equal powers in ϵ.

Applying the second condition in eq 16, $F_\epsilon(0) = 0$, to the eq 10-12 and noting that because of eq 17 the derivatives $w^{(n)}(x,0) = 0$, we obtain

$$Lw_\epsilon(x,0) = 0 \quad 0 \leqq x < \infty \tag{18}$$

where

$$L = D(\ )^{iv} + p(0)(\ )'' + \gamma \tag{19}$$

and the boundary conditions

$$w_\epsilon''(0,0) = 0 \tag{20}$$

$$Dw_\epsilon'''(0,0) + p(0)w_\epsilon'(0,0) = 0. \tag{21}$$

This is a linear eigenvalue problem for $w_\epsilon(x,0)$. It is identical to the formulation in the Euler method for obtaining p_{cr}.

This eigenvalue problem is satisfied by the trivial solution

$$w_\epsilon(x,0) \equiv 0 \tag{22}$$

which represents the continuation of the undeformed equilibrium branch. It is also satisfied, for the equilibrium branch which corresponds to $w(x,\epsilon) \neq 0$, by the *first eigenvalue (Rzhanitsyn 1955):*

$$p(0) = p_{cr} = \sqrt{\gamma D} \tag{23}$$

and the corresponding eigenfunction:

$$w_\epsilon(x,0) = Be^{-\alpha x}[\sin(\sqrt{3}\alpha x) - \sqrt{3}\cos(\sqrt{3}\alpha x)] \tag{24}$$

where B is an arbitrary constant and $\alpha = \sqrt[4]{\gamma/16D}$.

Next, we form $F_{\epsilon\epsilon}(0) = 0$. The resulting equations are the differential equation for $w_{\epsilon\epsilon}(x,0)$

$$Lw_{\epsilon\epsilon}(x,0) = -2p_\epsilon(0)w_\epsilon''(x,0) \quad 0 \leqq x < \infty \tag{25}$$

and the boundary conditions

$$w_{\epsilon\epsilon}''(0,0) = 0 \tag{26}$$

$$Dw_{\epsilon\epsilon}'''(0,0) + p(0)w_{\epsilon\epsilon}'(0,0) = -2p_\epsilon(0)w_\epsilon'(0,0) \tag{27}$$

where the differential operator L is given in eq 19. The above boundary value problem is satisfied for

$$w_\epsilon(x,0) \equiv 0 \quad \text{and} \quad w_{\epsilon\epsilon}(x,0) \equiv 0 \tag{28}$$

which represent the continuation of the undeformed equilibrium branch. For the equilibrium branch at the bifurcation point which corresponds to $w(x,\epsilon) \neq 0$, the operator L in eq 19 is singular [note that the corresponding homogeneous problem is identical to the one for $w_\epsilon(x,0)$]. Hence, a solution $w_{\epsilon\epsilon}(x,0)$ will exist only if the Fredholm alternative is satisfied.

In this connection, the nonhomogeneity in the second boundary condition is eliminated first, by introducing a new variable $v_1(x)$ as follows:

$$w_{\epsilon\epsilon}(x,0) = v_1(x) - \frac{2p_\epsilon(0)x}{p(0)} w_\epsilon'(0,0). \tag{29}$$

With this transformation, differential equation 25 becomes

$$Lv_1 = 2p_\epsilon(0)\left[\frac{\gamma x}{p(0)} w_\epsilon'(0,0) - w_\epsilon''(x,0)\right] \tag{25'}$$

and the boundary conditions in eq 26 and 27 become

$$v_1''(0) = 0 \tag{26'}$$

$$Dv_1'''(0) + p(0)v_1'(0) = 0. \tag{27'}$$

According to the Fredholm alternative, a solution $v_1(x)$, and hence $w_{\epsilon\epsilon}(x,0)$, will exist only if

$$\int_0^\infty p_\epsilon(0)\left[\frac{\gamma x}{p(0)} w_\epsilon'(0,0) - w_\epsilon''(x,0)\right]\psi(x)dx = 0 \tag{30}$$

where $\psi(x)$ is the nonzero solution of the homogeneous adjoint problem.

Because the operator is self-adjoint, it follows that

$$\psi(x) = w_\epsilon(x,0) \tag{31}$$

and the solvability condition (eq 30) reduces to

$$p_\epsilon(0)\int_0^\infty \left[\frac{\gamma x}{p(0)} w_\epsilon'(0,0) - w_\epsilon''(x,0)\right] w_\epsilon(x,0)\,dx = 0. \tag{32}$$

Noting that $w_\epsilon(x,0)$ is given in eq 24 and performing the integrations, it may be shown that the integral does not vanish. Hence eq 32 is satisfied when

$$p_\epsilon(0) = 0. \tag{33}$$

With eq 33, the boundary value problem for $w_{\epsilon\epsilon}(x,0)$ given by eq 25–27 reduces to the problem for $w_\epsilon(x,0)$ given by eq 18–21. Thus $w_{\epsilon\epsilon}(x,0)$ and $w_\epsilon(x,0)$ differ by an as yet undetermined coefficient β, namely,

$$w_{\epsilon\epsilon}(x,0) = \beta w_\epsilon(x,0). \tag{34}$$

To determine the next coefficients in eq 13 we form $F_{\epsilon\epsilon\epsilon}(0) = 0$. The resulting equations are the differential equation

$$Lw_{\epsilon\epsilon\epsilon}(x,0) = f(x) \quad 0 \leq x < \infty \tag{35}$$

where, noting that $p_\epsilon(0) = 0$,

$$f(x) = -[6D(4w_\epsilon' w_\epsilon'' w_\epsilon''' + w_\epsilon''^3 + w_\epsilon'^2 w^{iv}) + 9pw_\epsilon'^2 w_{\epsilon\epsilon}'' + 3p_{\epsilon\epsilon}w_\epsilon'']_{\epsilon=0} \tag{36}$$

and the boundary conditions

$$w_{\epsilon\epsilon\epsilon}''(0,0) = 0 \tag{37}$$

$$Dw_{\epsilon\epsilon\epsilon}'''(0,0) + p(0)w_{\epsilon\epsilon\epsilon}'(0,0) = -3\Big\{p_{\epsilon\epsilon}(0)w_\epsilon'(0,0) + D[w_\epsilon'^2(0,0)w_\epsilon'''(0,0) + 2w_\epsilon'(0,0)w_\epsilon''^2(0,0)]\Big\}. \tag{38}$$

The above equations are satisfied for

$$w_\epsilon(x) \equiv 0;\; w_{\epsilon\epsilon}(x,0) \equiv 0;\; w_{\epsilon\epsilon\epsilon}(x,0) \equiv 0 \tag{39}$$

which represent the continuation of the undeformed equilibrium branch.

For the deformed branch at the bifurcation point, the above equations constitute a nonhomogeneous boundary value problem with a singular operator. To facilitate the use of the Fredholm alternative, the nonhomogeneity in the second boundary condition is eliminated by introducing a new variable $v(x)$ as follows:

$$w_{\epsilon\epsilon\epsilon}(x,0) = v(x) - \frac{3\{\ldots\}x}{p(0)} \tag{40}$$

where $\{\ldots\}$ is from eq 38. With this transformation, differential equation 35 becomes

$$Lv = f(x) + \frac{3\gamma x}{p(0)}[p_{\epsilon\epsilon}(0)w_\epsilon'(0,0) + Dw_\epsilon'^2(0,0)w_\epsilon'''(0,0) + 2Dw_\epsilon'(0,0)w_\epsilon''^2(0,0)] \tag{35'}$$

and the boundary conditions of eq 37 and 38 become

$$v''(0) = 0 \tag{37'}$$

$$Dv'''(0) + p(0)v'(0) = 0. \tag{39'}$$

According to the Fredholm alternative, a solution $v(x)$, and hence $w_{\epsilon\epsilon\epsilon}(x,0)$, will exist only when

$$\int_0^\infty \left\{ f(x) + \frac{3\gamma x}{p(0)} \left[p_{\epsilon\epsilon}(0) w_\epsilon'(0,0) + D w_\epsilon'^2(0,0) w_\epsilon'''(0,0) + 2 D w_\epsilon'(0,0) w_\epsilon''^2(0,0) \right] \right\} \phi(x)\, dx = 0 \tag{41}$$

where

$$\phi(x) = w_\epsilon(x,0) \tag{31'}$$

is the nonzero solution of the homogeneous adjoint problem.

Performing the integrations indicated in eq 41, noting that $f(x)$ is given in eq 36 and $w_\epsilon(x,0)$ in eq 24, we obtain

$$p_{\epsilon\epsilon}(0) = -\frac{9}{28}\gamma B^2 \tag{42}$$

where B is an as yet undetermined coefficient. Thus, according to eq 13,

$$p(\epsilon) = p_{cr} - \frac{9}{56}\gamma B^2 \epsilon^2. \tag{43}$$

CONCLUSIONS

Equation 43 proves analytically, as predicted in *Simple model for ice cover buckling,* that the post-buckling branch of the semi-infinite plate with a "free" compressed edge exhibits values of p smaller than those of p_{cr}, even in the absence of lift-off. According to recent results by Plaut (1978) this may also occur for a floating plate strip "simply supported" and compressed along the straight parallel boundaries.

These findings indicate that for floating ice covers the critical buckling value p_{cr} should be used with caution. In this connection note that the analysis presented in *Semi-infinite floating plate with free edge* determines only the behavior of the post-buckling equilibrium branch in the vicinity of p_{cr}. The determination of the value p_L, when one exists, requires the solution of the nonlinear formulation based on eq 10.

During buckling of ice covers lift-off may take place. Therefore, the analysis for the determination of p_L should take this possibility into consideration. As shown in the model study, lift-off may strongly affect the p_L value and hence also the buckling load.

The parameter k^* in the model shown in Figure 2 represents the ratio of the stiffness of the base to the stiffness of the floating ice cover. For floating covers the stiffness of the liquid base is constant, whereas the plate stiffness reduces with decreasing thickness of the ice cover. Thus, a k^*-type value that corresponds to a thin floating plate will be much larger than the k^* value for a thick plate that is made of similar ice.

The post-buckling branch depends on the magnitude of the k^*-type value. Therefore, some buckling loads obtained from tests with very thin ice plates may not be suitable for the interpretation or prediction of the buckling response of thick ice covers. The results shown in Figure 5 also suggest that, when using thin plates in tests, special attention should be devoted to the possible occurrence of lift-off and its effect on the obtained results.

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